

# A L<sup>A</sup>T<sub>E</sub>X Package for Natural Deduction Proofs in the Jaśkowski and Kalish–Montague Styles

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## 1 Description

This package provides a L<sup>A</sup>T<sub>E</sub>X environment to typeset natural deduction proofs in format used by Jaśkowski (1934, footnote 3) or the format used by Kalish and Montague (1964) and Kalish, Montague, Mar (1980). This format differs from both the tree format initiated by Gerhard Gentzen (1934) and available in L<sup>A</sup>T<sub>E</sub>X with Sam Buss’s `bussproofs.sty` and from the more common Frederic Fitch style also available in L<sup>A</sup>T<sub>E</sub>X by either Johan Klüser’s `fitch.sty` or Peter Selinger’s `fitch.sty` (and more recently, from Richard Zach’s `lplfitch.sty`). A major point of difference between the various fitch-styles and the Jaśkowski and Kalish–Montague proofs is that these latter styles draw entire boxes or rectangles around subproofs, and these boxes can be embedded inside one another. Further details on both the logic and the history can be accessed from [1].

## 2 Downloads

Download the `natded.sty` and install in your L<sup>A</sup>T<sub>E</sub>X path.

## 3 Usage

After instructing the L<sup>A</sup>T<sub>E</sub>X compiler by using this package (by issuing a `\usepackage{natded}` at the beginning of the file, you can typeset natural deduction proofs in two major formats: Jaśkowski-style proofs are specified by `\Jproof{proof contents}` command while Kalish–Montague-style proofs are specified by `\KMproof{proof contents}` command. Note that both of these commands **should be used in math mode**. There are two major components for a proof:

1. Simple proof lines are specified by a `\proofline{formula}{annotation}` command,
2. Block commands, which initiate subproofs. These blocks (which normally contain lines and/or other blocks as *block contents*) come in two flavours:
  - (i) A Kalish–Montague block, where the conclusion comes before the block body:  
`\cbbk[guard symbol]{conclusion}{block contents}`,
  - (ii) A Jaśkowski block, where the conclusion comes after the block body:  
`\cabk[guard symbol]{block contents}{conclusion}`.
  - (iii) (In both of these commands, the block guard is optional as indicated by the brackets around the argument. The various roles of the guards are described in [1].

## 4 Comments on Usage

There are two distinct proof types – the Jaśkowski-proof, started by the `\Jproof` command, and the Kalish–Montague-proof, started by the `\KMproof` command. These overall proof commands require the remaining *proof contents* to be inside a set of braces. Each of these two proof-types require a single line as a conclusion, and can have many lines and many subproofs as *block contents*, and subproofs inside of them, and so forth.

The Jaśkowski-proofs have the conclusion come after the end of its justifying subproof block, whereas the Kalish-Montague-proofs have the conclusion before the start of its justifying subproof. Thus there are two separate commands that are in use: `\cablk` and `\cbblk`, standing for conclusion after and conclusion before, respectively. Obviously the `\cablk` is for use with the `\Jproof` while the `\cablk` is for use with the `\KMproof`.

The two arguments to each of the proof-styles are the conclusion and the block contents, each surrounded by a set of braces. Since the conclusions are always single formulas (possibly with a null annotation string), they are generated by indicating a formula: `\proofline{formula}{annotation}`, which is also the way to put a single formula on a line anywhere within a subproof. Note that `\proofline` requires each of its two arguments to be inside braces, but the line itself will not be within braces unless this is called for by some other feature (such as being the conclusion of one of block commands).

Note also that while the *formula* of a `\proofline` is automatically in math mode, the *annotation* is not. So, if reference to a logical symbol is required for the annotation, it must explicitly employ `$`.

## 5 Some Examples

For example, Figure 1 is generated by Listing 1, while Figure 2 is generated by Listing 2. Further examples, including their input listings, are in [1].

Listing 1: L<sup>A</sup>T<sub>E</sub>X code for Kalish–Montague-style proof

```

1 \[
2 \KMproof{
3   \cbblk{
4     \proofline{((P\rightarrow Q)\land (\neg R\rightarrow\neg Q))\rightarrow(P\rightarrow R)}{2--13
       Conditionalization}
5   }{
6     \proofline{((P\rightarrow Q)\land (\neg R\rightarrow\neg Q))}{Supposition}
7     \cbblk{
8       \proofline{(P\rightarrow R)}{4--13 Conditionalization}
9     }{
10    \proofline{P}{Supposition}
11    \proofline{((P\rightarrow Q)\land (\neg R\rightarrow\neg Q))}{2 Repeat}
12    \proofline{(P\rightarrow Q)}{5 Simplification}
13    \proofline{Q}{4, 6 Modus Ponens}
14    \proofline{(\neg R\rightarrow\neg Q)}{5 Simplification}
15    \cbblk{
16      \proofline{R}{10--13 Reductio ad Absurdum}
17    }{
18      \proofline{\neg R}{Supposition}
19      \proofline{(\neg R\rightarrow\neg Q)}{8 Repeat}
20      \proofline{\neg Q}{10, 11 Modus Ponens}
21      \proofline{Q}{7 Repeat}
22    }
23  }
24 }
25 ]
26 \]

```

Listing 2: L<sup>A</sup>T<sub>E</sub>X code for Jaśkowski-style proof

```

1 \[
2 \Jproof{
3   \cablk{
4     \proofline{((P\rightarrow Q)\land (\neg R \rightarrow\neg Q))}{Supposition}
5   \cablk{
6     \proofline{P}{Supposition}
7     \proofline{((P\rightarrow Q)\land (\neg R\rightarrow\neg Q))}{1 Repeat}
8     \proofline{(P\rightarrow Q)}{3 Simplification}
9     \proofline{Q}{2, 4 Modus Ponens}
10    \proofline{(\neg R\rightarrow\neg Q)}{3 Simplification}
11    \cablk{
12      \proofline{\neg R}{Supposition}
13      \proofline{(\neg R\rightarrow\neg Q)}{6 Repeat}
14      \proofline{\neg Q}{7, 8 Modus Ponens}
15      \proofline{Q}{5 Repeat}
16    }{

```

Figure 1: A Kalish–Montague-style proof

1.	<i>Show</i> $((P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q)) \rightarrow (P \rightarrow R)$	2–13 Conditionalization
2.	$((P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q))$	Supposition
3.	<i>Show</i> $(P \rightarrow R)$	4–13 Conditionalization
4.	$P$	Supposition
5.	$((P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q))$	2 Repeat
6.	$(P \rightarrow Q)$	5 Simplification
7.	$Q$	4, 6 Modus Ponens
8.	$(\neg R \rightarrow \neg Q)$	5 Simplification
9.	<i>Show</i> $R$	10–13 Reductio ad Absurdum
10.	$\neg R$	Supposition
11.	$(\neg R \rightarrow \neg Q)$	8 Repeat
12.	$\neg Q$	10, 11 Modus Ponens
13.	$Q$	7 Repeat

Figure 2: A Jaśkowski-style proof

1.	$((P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q))$	Supposition
2.	$P$	Supposition
3.	$((P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q))$	1 Repeat
4.	$(P \rightarrow Q)$	3 Simplification
5.	$Q$	2, 4 Modus Ponens
6.	$(\neg R \rightarrow \neg Q)$	3 Simplification
7.	$\neg R$	Supposition
8.	$(\neg R \rightarrow \neg Q)$	6 Repeat
9.	$\neg Q$	7, 8 Modus Ponens
10.	$Q$	5 Repeat
11.	$R$	7–10 Reductio ad Absurdum
12.	$(P \rightarrow R)$	2–11 Conditionalization
13.	$((P \rightarrow Q) \wedge (\neg R \rightarrow \neg Q)) \rightarrow (P \rightarrow R)$	1–12 Conditionalization

```

17     \proofline{R}{7--10 Reductio ad Absurdum}
18   }
19   }{
20   \proofline{(P\rightarrow R)}{2-11 Conditionalization}
21   }
22   }{
23   \proofline{(((P\rightarrow Q)\land (\neg R\rightarrow\neg Q))\rightarrow(P\rightarrow R))}{1--12
      Conditionalization}
24   }
25   }
26   \]

```

## References

- [1] Mohammad M. Ajallooeian and Francis Jeffrey Pelletier. *Kalish/Montague and Jaśkowski Natural Deduction*. *L<sup>A</sup>T<sub>E</sub>X Package Manual on CTAN*. 2014.